Multifractal Omori-Utsu Law for Earthquake Triggering: New Tests on the Harvard and Japanese Catalogs

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The Multifractal Stress-Activated (MSA) model is a statistical model of triggered seismicity based on mechanical and thermodynamic principles. It predicts that, above a triggering magnitude cut-off M_0, the exponent p of the Omori-Utsu law for the seismic decay of aftershocks is a linear increasing function $p(M) = aM + b$ of the mainshock magnitude M [*Ouillon and Sornette*, 2005]. First empirical support for this prediction has been presented for the Southern California SCEC catalog. Here, we confirm this law on the worlwide Harvard CMT and the Japanese JMA catalogs, with similar ranges of variation from $p(M = 3) = 0.7 \pm 0.1$ to $p(M = 8) = 1.1 \pm 0.2$. However, the statistically significant differences of the slopes a, intercepts b and cut-offs M_0 suggest different multifractal properties of the three catalogs, likely associated with different thermal and mechanical properties.

1. Introduction

The popular concept of triggered seismicity reflects the growing consensus that earthquakes interact through a variety of fields (elastic strain, ductile and plastic strains, fluid flow, dynamical shaking and so on). The concept of triggered seismicity was first introduced from mechanical considerations, by looking at the correlations between the spatial stress change induced by a given event (generally quoted as a mainshock), and the spatial location of the seismicity that appeared to be temporally correlated with, and following, that main event (the so-called aftershocks) [*King et al.*, 1994; *Stein*, 2003]. Complementarily, purely statistical models have been introduced to take account of the fact that the main event is not the sole event to trigger some others, but that aftershocks may also trigger their own aftershocks and so on. Those models, of which the ETAS (Epidemic Type of Aftershock Sequences) model [*Kagan and Knopoff*, 1981; *Ogata*, 1988] is a standard representative with good explanatory power [*Saichev and Sornette*, 2006], unfold the cascading structure of earthquake sequences. This class of models show that real-looking seismic catalogs can be generated by using a parsimonious set of parameters.

Very few efforts have been devoted to bridge both approaches, so that a statistical mechanics of seismicity based on physical principles could be built. Dieterich [1994] has considered both the spatial complexity of stress increments due to a main event and one possible physical mechanism that may be the cause of the time-delay in the aftershock triggering, namely state-and-rate friction. Dieterich's model predicts that aftershocks sequences decay with time as t^{-p} with $p \simeq 1$ independently of the mainshock magnitude, a value which is often observed but only for sequences with a sufficiently large number of aftershocks triggered by large earthquakes, typically for main events of magnitude 6 or larger. Dieterich's model has in particular the drawback of neglecting the stress changes due to the triggered events themselves and cannot be considered as a consistent theory of triggered seismicity.

Recently, two of us [*Ouillon and Sornette*, 2005; *Sornette and Ouillon*, 2005] have proposed a simple physical model of self-consistent earthquake triggering, the Multifractal Stress-Activated (MSA) model, which takes into account the whole deformation history due to seismicity. This model assumes that rupture at any scale is a thermally activated process in which stress modifies the energy barriers. This formulation is compatible with all known models of earthquake nucleation, and in particular contains the state-and-rate friction mechanism as a particular case. At any given place in the domain, the seismicity rate λ is given by $\lambda(t) = \lambda_0 \exp(\sigma(t)/\sigma_T)$, where $\sigma(t)$ is the total local stress at time t and $\sigma_T = kT/V$ is an activation stress defined in terms of the activation volume V and an effective temperature T (k is the Boltzmann constant). Among others, Ciliberto et al. [2001] and Saichev and Sornette [2005] have shown that the presence of frozen heterogeneities, always present in rocks and in the crust, has the effect of renormalizing and amplifying the temperature through the cascade of micro-damage to the macro-rupture, while conserving the same Arrhenius structure of the activation process. The prefactor λ_0 depends on the loading rate and the local strength. The domain is considered as elasto-visco-plastic with a large Maxwell time τ_M. For $t < \tau_M$, the model assumes that the local stress relaxes according to $h(t) = h_0/(t + c)^{1+\theta}$, where c is is a small regularizing time scale. The local stress $\sigma(t)$ depends on the loading rate at the boundaries of the domain and on the stress fluctuations induced by all previous events that occurred within that domain. At any place, any component s of the stress fluctuations due to previous events is considered to follow a power-law distribution $P(s)ds = C/(s^2 + s_0^2)^{(1+\mu)/2}ds$. For $\mu(1 + \theta) \simeq 1$, Ouillon and Sornette [2005] found that (i) a magnitude M event will be followed by a sequence of aftershocks which takes the form of an Omori-Utsu law with exponent p, (ii) this exponent p depends linearly on the magnitude M of the main event and (iii) there exists a lower magnitude cut-off M_0 for mainshocks below which they do not trigger. In contrast with the phenomenological statistical models such as the ETAS model, the MSA model is based on firm mechanical and thermodynamical principles.

Ouillon and Sornette [2005] has tested this prediction on the SCEC catalog over the period from 1932 to 2003. Using a superposed epoch procedure to stack aftershocks series triggered by events within a given magnitude range, they found that indeed the p-value increases with the magnitude M of the main event according to $p_S(M) = a_S M + b_S =$

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$a_S(M - M_{0,S})$, where $a_S = 0.10 \pm 0.01(1\text{std}), b_S = 0.37 \pm 0.06(1\text{std}), M_{0,S} = -3.5 \pm 1.0(1\text{std})$. The error bars are obtained by using a bootstrap technique described below, which allows us to show that the hypothesis that $a_S = 0$ can be rejected with a confidence level close to 100%, confirming that there exists a very significant increasing linear relationship between p and M for earthquakes recorded in the SCEC catalog. Performing the same analysis on synthetic catalogs generated by the ETAS model for which p is by construction independent of M did not show an increasing $p(M)$, suggesting that the results obtained on the SCEC catalog reveal a genuine multifractality which is not biased by the method of analysis.

Here, we extend the analysis to other areas in the world (the worlwide Harvard CMT catalog and the Japanese JMA catalog), to put to test again the theory and to check whether the parameters a and b are universal or on the contrary vary systematically from one catalog to the other, perhaps revealing meaningful physical differences between the seismicity of different regions.

2. The worlwide Harvard CMT and the Japanese JMA catalogs

The worldwide CMT Harvard catalog used here goes from January 1977 to December 2003 inclusive. This catalog is considered to be complete for events of magnitude 5.5 or larger. We thus deleted events below this threshold before searching for the aftershocks. Due to the rather small number of events in this catalog, we did not impose any limit on the depth of events.

The JMA catalog used here covers a much longer period from May 1923 to January 2001 inclusive. We restricted our analysis to the zone (130°E to 145°E in longitude and 30°N to 45°N in latitude), so that its northern and eastern boundaries fit with those of the catalog, while the southern and eastern boundaries fit with the geographic extension of the main japanese islands. This choice selects the earthquakes with the best spatial location accuracy, close to the inland stations of the seismic network. In our analysis, the mainshocks are taken from this zone and in the upper 70km, while we take into account their aftershocks which occur outside and at all depths.

Our detailed analysis of the aftershock series at spatial scales down to $20km$ uncovered a couple of zones where large as well as small main events are not followed by the standard Omori-Utsu power-law relaxation of seismicity. The results concerning these zones will be presented elsewhere. Here, we simply removed the corresponding events from the analysis. The geographical boundaries of these two anomalous zones are $[130.25°\text{E}; 130.375°\text{E}] \times [32.625°\text{N}; 32.75°\text{N}]$ for the first zone, and $[138.75°\text{E}; 139.5°\text{E}] \times [33°\text{N}; 35°\text{N}]$ for the second one (the so-called Izu islands area). This last zone is particularly known to be the locus of earthquakes swarms which may explain the observed anomalous aftershock relaxation. We have been conservative in the definition of this zone along the latitude dimension so as to avoid possible contamination in the data analysis which would undermine the needed precise quantification of the p-values.

The completude of the JMA catalog is not constant in time, as the quality of the seismic network increased more recently. We computed the distribution of event sizes year by year, and used in a standard way [*Kagan*, 2003] the range over which the Gutenberg-Richter law is reasonably well-obeyed to infer the lower magnitude of completeness. For our analysis, we smooth out the time dependence of the magnitude threshold M_c above which the JMA catalog can be considered complete from roughly $M_c(1923) = 6$, to $M_c(1930 - 1960) = 5$, $M_c(1960 - 1990) = 4.5$ with a final progressive decrease to $M_c = 2.5$ for the most recent past. This time-dependence of the threshold $M_c(t)$ will be used for the selection of mainshocks and aftershocks.

3. Methodology of the multifractal analysis

3.1. Step 1: selection of aftershocks

We follow the same method to construct stacked aftershocks time series as in [*Ouillon and Sornette*, 2005]. Briefly, all earthquakes in the catalog are considered successively as potential mainshocks. For each event, we look at the subsequent seismicity within $T = 1$ year and within a distance $R = 2L$, where L is the rupture length of the mainshock, which is determined empirically from the magnitude using Wells and Coppersmith [1994]'s relationship. If the rupture length is smaller than the spatial location accuracy (which we assume here to be 10km), we set $L = 10km$. If an event has previously been tagged as an aftershock of a larger event, then it is removed from the list of potential mainshocks. Aftershock series are then sorted according to the magnitude of the main event, and stacked using a superposed epoch procedure within given mainshock magnitude ranges. As for the SCEC catalog, we choosed mainshock magnitude intervals to vary by half-unit magnitude steps.

For the JMA catalog, we take into account the variation of $M_c(t)$ as follows. Individual aftershock times series were considered in the stack only if the magnitude of the main event, occurring at time t_0, was larger than $M_c(t_0)$. If this main event obeys that criterion, only its aftershocks above $M_c(t_0)$ are considered in the series. This methodology allowed us to use the maximum amount of data with sufficient accuracy to build our staked time series of aftershock decay rates.

3.2. Step 2: fitting procedure of the stacked time series

Once aftershocks series have been selected and stacked, we fit binned data with $N(t) = A \cdot t^{-p} + B$, which includes a constant background rate B. Here, $N(t)$ is the rate of triggered seismicity at time t after a mainshock that occured at $t = 0$. The time axis is binned in intervals according to a geometrical series so that the width of the time intervals grows exponentially with time. We then simply count the number of aftershocks contained within each bin, then divide this number by the linear size of the interval to obtain the rate N. The fitting parameters A, B, p are then obtained by a standard grid search.

Note that, as the linear density of bins decreases as the inverse of time, each bin receives a weight proportional to time, balancing the weight of data points along the time axis. In our binning, the linear size of two consecutive intervals increases by a factor $r > 1$. Since the choice of r is arbitrary, it is important to check for the robustness of the results with respect to r. We thus performed fits on time series binned with 20 different values of r, from $r = 1.1$ to $r = 3$ by step of 0.1. We then checked whether the fitted parameters A, B and p were stable with r. We also computed the average values and standard deviations of all fitting parameters over the 20 r values. We excluded the early times, where aftershock catalogs appear to be incomplete [*Kagan*, 2004]. Finally, a p-value determined within the magnitude interval $[M_1; M_2]$ for the mainshock was associated with magnitude $\frac{M_1+M_2}{2}$.

3.3. Step 3: Regressions in the (M, p) space and tests of significance

The next step consists in performing a standard linear regression in the (M, p) plane, in order to determine the validity of the prediction $p(M) = aM + b$ or, equivalently, $p(M) = a(M - M_0)$, with $M_0 = -b/a$. For each catalog, we have tested the significance of the estimations obtained for a, b and M_0 against a series of null hypotheses defined as follows : (i) the slope a is not significantly different from 0; (ii) and (iii) the slopes a and intercepts b are the same from one catalog to another; (iv) the cutoff magnitudes M_0 are the same from one catalog to another.

Such tests are usually performed through the use of $F-$ or t-tests. The statistical confidence obtained with these tests are reliable only if the residues (defined as the difference between the observed (M, p) values and their linear regression $p(M) = aM + b$) are Gaussian so that standard asymptotic theorems apply. This is doubtful for our data and we therefore used a bootstrap method which circumvents these conditions. The bootstrap approach is performed by first considering a seismic catalog, from which we retrieve n pairs (M_i, p_i), with $i = 1, ..., n$. We first perform a linear fit on this data set and obtain the corresponding a, b and M_0 fitted parameters, as well as the residues $R_i = p_i - aM_i - b$, $i = 1, ..., n$. We then reshuffle the $R_i's$ at random to obtain a new ordered set of residues $(M_i, R_{i,r})$. This allows us to build a new synthetic data set $(M_i, p_{i,r})$, with $p_{i,r} = aM_i + b + R_{i,r}$. We then perform the linear regression $p_r(M) = a_r M + b_r = a_r(M - M_{0,r})$ on the data set constituted of the n pairs $(M_i, p_{i,r})$. We perform this reshuffling/refitting procedure 10^4 times. By storing the 10^4 values a_r, b_r and $M_{0,r}$, the cumulative probability distribution for a_r, b_r and $M_{0,r}$ is estimated, from which the confidence levels for the above hypotheses (i)-(iv) can be obtained. For example, consider a probability level $q_0 < \frac{1}{2}$. If an arbitrary value a_0 is smaller than the quantile q_0 or larger than the quantile $1 - q_0$ of the cumulative probability function of a_r, then the hypothesis that the slope a is a_0 can be rejected with a confidence level larger than or equal to $1 - 2q_0$. For the present analysis, we consider 95% confidence levels in all our tests, namely $2q_0 = 0.05$.

4. Results

For the Harvard catalog, six magnitude intervals were used from [5.5; 6] to [8; 8.5]). Figure 1 shows the six stacked

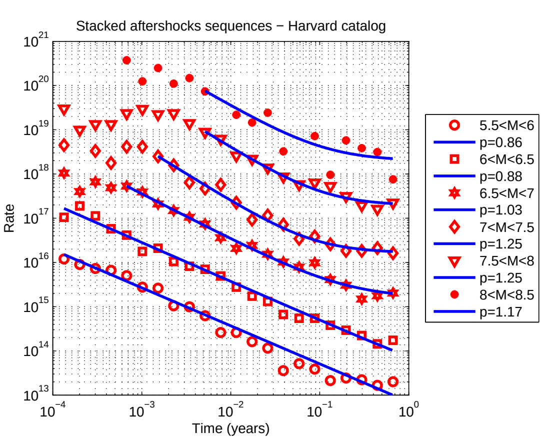

Figure 1. Seismic decay rates of stacked sequences for six magnitude intervals of the mainshocks in the Harvard catalog, obtained with the geometrical ratio $r = 1.5$ for the binning of time intervals. Each data set for a given magnitude interval $[M_1, M_1 + 0.5]$ has been translated vertically by multiplying the rates by 10^{2M_1}.

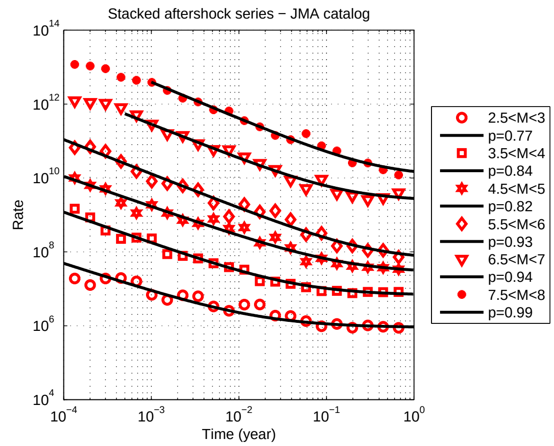

Figure 2. Same as Figure 1 for the JMA catalog.

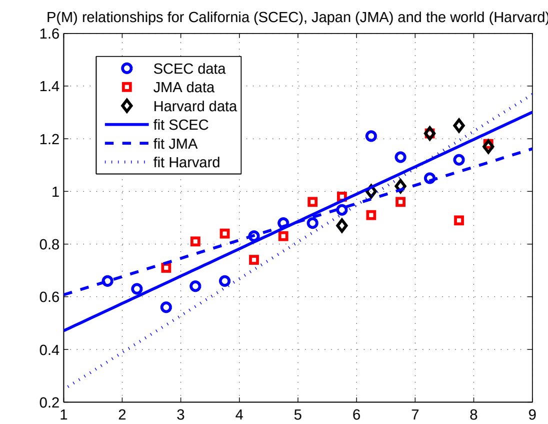

Figure 3. Exponent p averaged over the 20 values r as a function of the middle value of the corresponding magnitude interval for the SCEC catalog (taken from [*Ouillon and Sornette*, 2005]) the Harvard and JMA catalogs (this study). Their linear regressions are shown as straight lines.

aftershocks time series and their fits with expression $N(t) = A \cdot t^{-p} + B$ for the value $r = 1.5$ of the geometrical ratio used for the binning of time intervals. Similar curves and results are obtained for the 20 other values of r, confirming the robustness of the analysis. Figure 3 plots the exponent p averaged over the 20 values r as a function of the middle value of the corresponding magnitude interval. These values and associated standard deviations are: $p(5.75) = 0.87 \pm 0.05$, $p(6.25) = 1.00 \pm 0.09$, $p(6.75) = 1.02 \pm 0.05$, $p(7.25) = 1.22 \pm 0.14$, $p(7.75) = 1.25 \pm 0.09$, $p(8.25) = 1.17 \pm 0.29$. The exponents $p(M)$ obtained for the Harvard catalog are close to those obtained for the SCEC catalog for the magnitudes which are common to the two catalogs. The regression of p as a function of M according to $p_H(M) = a_H M + b_H = a_H(M - M_{0,H})$ yields $a_H = 0.14 \pm 0.03, b_H = 0.11 \pm 0.23, M_{0,H} = -1.28 \pm 2.21$. The rather large standard deviations on b_H and $M_{0,H}$ result from the relatively narrow magnitude range available for the fit.

For the JMA catalog, 12 magnitude intervals were used from [2.5; 3] to [8; 8.5]). Figure 2 shows the 12 stacked aftershocks time series and their fits for $r = 1.5$. Figure 3 plots the exponent p averaged over the 20 values r as a function of the middle value of the corresponding magnitude interval. These values and associated standard deviations are: $p(2.75) = 0.71 \pm 0.05$, $p(3.25) = 0.81 \pm 0.06$, $p(3.75) = 0.84 \pm 0.03$, $p(4.25) = 0.74 \pm 0.03$, $p(4.75) = 0.83 \pm 0.04$, $p(5.25) = 0.96 \pm 0.04$, $p(5.75) = 0.98 \pm 0.09$, $p(6.25) = 0.91 \pm 0.05$, $p(6.75) = 0.96 \pm 0.09$, $p(7.25) = 1.22 \pm 0.07$, $p(7.75) = 0.89 \pm 0.04$, $p(8.25) = 1.18 \pm 0.13$. The values of p are again similar to those obtained for the Harvard and SCEC catalogs. The regression of p as a function of M according to $p_J(M) = a_J M + b_J = a_J(M - M_{0,J})$ yields $a_J = 0.07 \pm 0.02, b_J = 0.54 \pm 0.09, M_{0,J} = -8.6 \pm 3.8$. Note that the standard deviation of $M_{0,J} = -b_J/a_J$ is amplified by the smallness of a_J.

The linear dependence $p(M) = aM + b$ predicted by the MSA model provides a good fit to all three data sets (SCEC, Harvard and JMA catalogs). Our statistical significance tests using 10^4 bootstraps shows that all coefficients a_S, a_H and a_J are significantly different from 0 at confidence levels very close to 100%, confirming empirically the main novel prediction of the MSA model that the Omori-Utsu p-value is not a universal constant but increases with the magnitude of the mainshock magnitude. We also tested whether the coefficients a_S, a_H and a_J could be considered equal, given the empirical noises and the uncertainties introduced by the analysis. For this, we tested the six possibilies represented symbolically by $(a_S \to a_H; a_H \to a_S; a_S \to a_J; a_J \to a_S; a_H \to a_J; a_J \to a_H)$ that the data for one catalog could be explained by the coefficient a of another catalog. Using again the bootstrap method to compare the value a of one catalog with the statistical bootstrap ensemble generated with that of another catalog leads to reject five of the six possibilities at the 95% confidence level. But we can not reject the hypothesis that the (M, p) values for the Harvard catalog are compatible with a slope a_H equal to the slope a_S of the SCEC catalog. Similar tests performed by the intercept coefficients b yield similar results that all b's are significantly different, with again one exception: for the Harvard catalog, we could not reject the hypothesis that its data set of (M, p) pairs may have been generated by a $p(M)$-relationship with the same intercept as the one computed for the SCEC catalog or for the JMA catalog. The tests on the a and b coefficients are not independent but similar tests for the lower magnitude cut-off M_0 nevertheless yield the same conclusion: all M_0-values are significantly different from one set to another at the 95% confidence level for any pair of catalogs, except that we could not reject the hypothesis that the Harvard (M, p) data may have been generated by a relationship $p(M)$ with the same M_0 as the one obtained for the SCEC catalog.

The statistically significant differences of the slopes a and intercepts b reflected in the different lower magnitude cutoffs M_0 suggest different multifractal properties of the three catalogs associated with different minimum triggering sizes. Converted to rupture lengths L_0 using [*Wells and Coppersmith*, 1994], we obtain $L_{0,S} = 4cm \pm 5cm$ for the SCEC catalog, $L_{0,H} = 2mm \pm 5mm$ for the Harvard catalog and $L_{0,J} = 4m \pm 6.5m$ for the JMA catalog. We conjecture that these different values are likely associated with different thermal and mechanical properties of the distinct world regions.

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